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Studies of levels in ^{29}P up to 4·76 MeV using the ^{28}Si(p, γ)^{29}P reaction

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Abstract. Properties of seven of the first eight excited states in ^{29}P have been obtained from studies of the proton capture gamma radiation resonances at 720, 1381, 1652 and 2086 keV. Spin–parity assignments, branching ratios and multipole mixing ratios are deduced from angular distributions and lifetimes found from Doppler shift attenuation measurements. New measurements reported here are: 2423 keV level, lifetime of 70 ± 40 fs; 3450 keV level, $\Gamma_\gamma \leqslant 60$ μeV; 4344 level, $\delta(2{\cdot}96) = 0{\cdot}04$; 4761 keV level, decays 90% to GS, 7% to 1·38 MeV level, 3% to 2·424 MeV level. There is evidence to suggest that the 4761 keV level is a doublet.

1. Introduction

The nucleus ^{29}P is interesting for study because of its position near the centre of the 2s–1d shell, a nuclear mass region which has attracted several different model descriptions in an attempt to explain the features of nuclei where the deformation appears to be changing from prolate to oblate. The mirror nucleus ^{29}Si has been represented in terms of the Nilsson model by Bromley *et al* (1957) who found that the then known experimental properties could be reproduced reasonably well by assuming a small negative deformation. Ejiri (1964) later performed a similar calculation for ^{29}P and reproduced the experimental energy level spectrum up to about 4·5 MeV by proposing that the ground and the first excited states are the $K = \frac{1}{2}$ (orbit 9) and $K = \frac{3}{2}$ (orbit 8) band heads. Ejiri also performed a weak coupling model calculation for ^{29}P, as have Bailey and Choudhury (1970) but these were rather less successful in reproducing the energy level spectrum, although more recently Castel *et al* (1970), by inclusion of the effect of hole states, improved the predictions for the properties of ^{29}Si.

The experimental information on the lower energy levels of ^{29}P is still somewhat sparse compared with the situation for ^{29}Si, which has been investigated in great detail recently (Baker and Segal 1968, Becker *et al* 1967, Denhard and Yntema 1970, Main *et al* 1970). In particular, γ decay has never been observed from the level most likely to be the mirror of the $\frac{7}{2}^-$ fifth excited state of ^{29}Si and gamma decay of several other states has only been investigated with NaI detectors.

In the present experiments information has been gained on seven of the first eight excited states of ^{29}P with the aid of resonances in the ^{28}Si(p, γ)^{29}P reaction at proton energies of 720, 1381, 1652 and 2086 keV. Although studies of the $E_p = 1381$ keV resonance have been reported several times recently (Monahan *et al* 1970, Williams *et al* 1970, Broude *et al* 1969) our results for this resonance are included both for comparison and because some additional information was gained.

2. Experimental methods

The experiments were performed with the aid of the University of Manchester 6 MV Van de Graaff accelerator, which produced analysed proton beams of up to 15 μA at the target which was water cooled. The target material was evaporated onto thin gold foil which had been fixed to copper strips by silver paste. The target chamber system, which is described in more detail by Barnes (1971), allowed a detector-to-target-spot distance of 4 cm to be used over an angular range of 0° to 120°.

Three configurations of γ ray detectors were used. A 7·5 cm × 7·5 cm NaI detector, mounted at 145° underneath the beam line was used only for NaI–Ge(Li) coincidence measurements. One Ge(Li) detector was fixed at approximately 90° to the beam and was used as a monitor during angular distribution experiments. The use of a monitor of similar resolution to the movable detector has proved to be an important factor in ensuring reliable and consistent normalizations. A second Ge(Li) detector was mounted on a turntable whose axis of rotation passed through the centre of the target chamber. Four different Ge(Li) detectors† were used during these experiments with volumes from 26 cm^3 to 70 cm^3 and resolutions between 2·0 and 2·3 keV at 1·33 MeV.

The absolute efficiencies of these detectors were measured up to 1·836 MeV by placing small calibrated sources in the centre of the target chamber. The calibration was extended up to 3·25 MeV by measuring the known intensities of a ^{56}Co source and up to 5 MeV which was the highest γ ray energy investigated, with the aid of the ^{27}Al(p, γ) ^{28}Si reaction at a proton energy of 2·489 MeV. This produces a cascade sequence with 100 % branches giving γ rays of 9·46, 2·83 and 1·78 MeV.

The isotropy of the system was measured by placing point γ ray sources of different energy over the beam mark on the target after each angular distribution measurement. The anisotropy was always less than 5 %. The angular distribution attenuation coefficients for the Ge(Li) detectors were measured by placing a small ^{22}Na source on the target spot and then recording the events in the Ge(Li) detector in coincidence with γ rays detected in a 7·5 cm × 7·5 cm NaI detector positioned approximately 1 m from the target. This has the effect of scanning a collimated beam of 0·511 MeV γ rays across the face of the detector as it is rotated about the target chamber. The resulting plot of efficiency against angle was then integrated numerically according to the expression of Ferguson (1965) to calculate the Q_k coefficients.

The data were digitized‡ to 4096 channel accuracy or to 8192 channel accuracy when precise energy measurements were required and recorded by a DEC PDP 7 on-line computer system.

The parameters determined from the angular distribution measurements, namely the spins of initial and final states, the multipole mixing ratio and the normalization coefficient, were found by the usual method of finding the minimum of

$$\chi^2 = \sum_{i=1}^{n} (V(\theta) - A_0 W(\theta_i))^2 \frac{1}{E_i^2}.$$

A_0 is the normalization coefficient, $V(\theta)$ the experimental angular ditribution, E the experimental error and $W(\theta)$ the theoretical distribution, calculated using assumed values for the spin and mixing ratio. Values of these parameters which produced a χ^2 value larger than the statistical 0·1 % confidence level were then rejected as being unlikely.

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† Supplied by Ortec Inc, Oak Ridge, Tennessee, USA.

‡ ADC Northern Scientific type 626.

In this way an angular distribution measurement usually produced a unique value for the spin sequence and either one or two values for the mixing ratio δ, which corresponded to minima in the plot of χ^2 against δ for fixed spin values.

As a result of δ entering nonlinearly into the theoretical expression for the angular distribution there has been confusion in the past as to the error of the extracted parameter. The methods of Cline and Lesser (1970) and Bevington (1969), which are semi-analytical and depend on the assumption that the χ^2 against δ curve is quadratic over the range of interest and the graphical method demonstrated, for example, by Taras (1970) were compared for several samples of the data and found to produce similar results. The errors on δ quoted in the present results were derived by a technique equivalent to the graphical method and are therefore independent of arbitrary confidence level considerations. The sign convention used for δ is that of Rose and Brink (1967).

3. Experimental details and results

3.1. *The fifth excited state*

The level has been observed via the ^{28}Si(d, n) ^{29}P reaction (Davies *et al* 1966), the ^{32}S(p, α) ^{29}P reaction (Ejiri *et al* 1964b) and the ^{28}Si(^{3}He, d) ^{29}P reaction (Ejiri *et al* 1966), but its gamma decay has never been found. These reactions gave a mean excitation energy of 3·446 ± 0·016 MeV for the state and suggest a mainly single particle character and negative parity for the level.

A knowledge of the electromagnetic decay modes would obviously be of interest particularly in view of the recent measurement of the J^π of the corresponding level in ^{29}Si as $\frac{7}{2}^-$ by a technique which is independent of reaction models (Spear *et al* 1971).

Van Oostrum *et al* (1961), Ejiri *et al* (1964a) and recently Zuk *et al* (1971) have searched for the level using the ^{28}Si(p, γ) ^{29}P reaction without success using the usual NaI detector yield curve method. The approach used in the present work was to try to utilize the high resolution of the Ge(Li) detector by accumulating spectra taken at several different proton energies within the range of interest, to see if any of the gamma ray lines present in the spectra could be identified as being due to ^{29}P. A 200 μg cm^{-2} thick natural silicon target was used, giving a proton energy loss of approximately 45 keV within the target, and singles spectra were accumulated at proton energies within the range 680 keV to 765 keV in steps of approximately 25 keV.

By comparison with the decay of the mirror level at 3·623 MeV in ^{29}Si (Endt and Van der Leun 1967), we would expect the transition from the 3·45 MeV level to the second excited state at 1·954 MeV and the subsequent transition to the ground state to be the most favoured means of gamma decay.

Neither of these lines was observed in any of the spectra, nor was any other γ ray line that could possibly be involved in the decay of the fifth excited state. The products of the 693 keV resonance in the ^{29}Si(p, γ) ^{30}P reaction were observed at a proton energy of 710 keV, thus confirming that the detection system was working properly and that the proton energy was reliably known.

From the empirical data an upper limit for Γ_γ was calculated. It was assumed that the spin of the level is $\frac{7}{2}$, that is, the same as the mirror level in ^{29}Si and the decay goes 100% to the 1·954 MeV state (in ^{29}Si the equivalent branch is 90%).

Using these assumptions we obtain $\Gamma_\gamma < 6 \times 10^{-5}$ eV. This improves upon van Oostrum's value of $\Gamma_\gamma < 5 \times 10^{-4}$ eV and the value of $\Gamma_\gamma < 9 \times 10^{-5}$ eV (assuming $\Gamma_p \gg \Gamma_\gamma$) of Zuk *et al* (1971).

3.2. The $E_p = 1650$ keV resonance

This is a broad strong resonance which has been investigated several times with the aid of NaI detectors (see eg van Oostrum 1961, Ejiri *et al* 1964a, Newton 1960). The Ge(Li) spectra were taken in singles and in coincidence with a 1·3–3·5 MeV window on the γ ray spectrum from a 7·5 cm × 7·5 cm NaI detector. These spectra confirmed that the branch through the first excited state is the only alternative mode of decay to the ground state transition.

The angular distribution of the intense 4·34 MeV γ rays was measured at seven angles in the range 0° to 90°. This confirmed the pure E1 nature of the transition from the $\frac{3}{2}^-$ resonance level to the ground state (Newton 1960, Ejiri *et al* 1964a).

The angular distribution of the 2·96 MeV γ rays has not been reported previously and is shown in figure 1. The angular distributions of the 2·69 and 1·38 MeV γ rays considered jointly are consistent only with the assigned $J = \frac{3}{2}$ value for the first excited state (Endt and Van der Leun 1967) and consequently values for $\delta(2{\cdot}96)$ of $-4{\cdot}7$ or 0·04. The lifetime of the first excited state (see § 3.6) gives a strong argument for positive parity of the state and thus eliminates the $\delta(2{\cdot}96) = -4{\cdot}7$ value, which would be inconsistent with E1, M2 mixing, so that $\delta(2{\cdot}96) \leqslant 0{\cdot}04$.

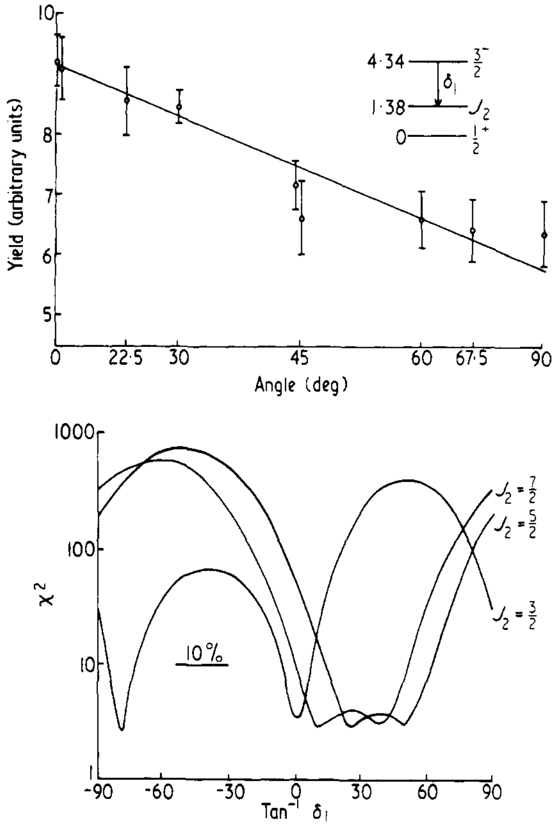

Figure 1. Angular distribution and χ^2 against δ plot of the γ rays from the transition from the seventh excited state to the first excited state at the $E_p = 1{\cdot}65$ MeV resonance.

3.3. The E_p = 1381 keV resonance

A 30 μg cm^{-2} thick natural silicon target was used to produce the spectrum shown in figure 2(*a*), which confirms that the 4·08 MeV state decays only to the first and second excited states, giving γ rays of 2·696 MeV with 1·384 MeV and 2·129 MeV with 1·954 MeV

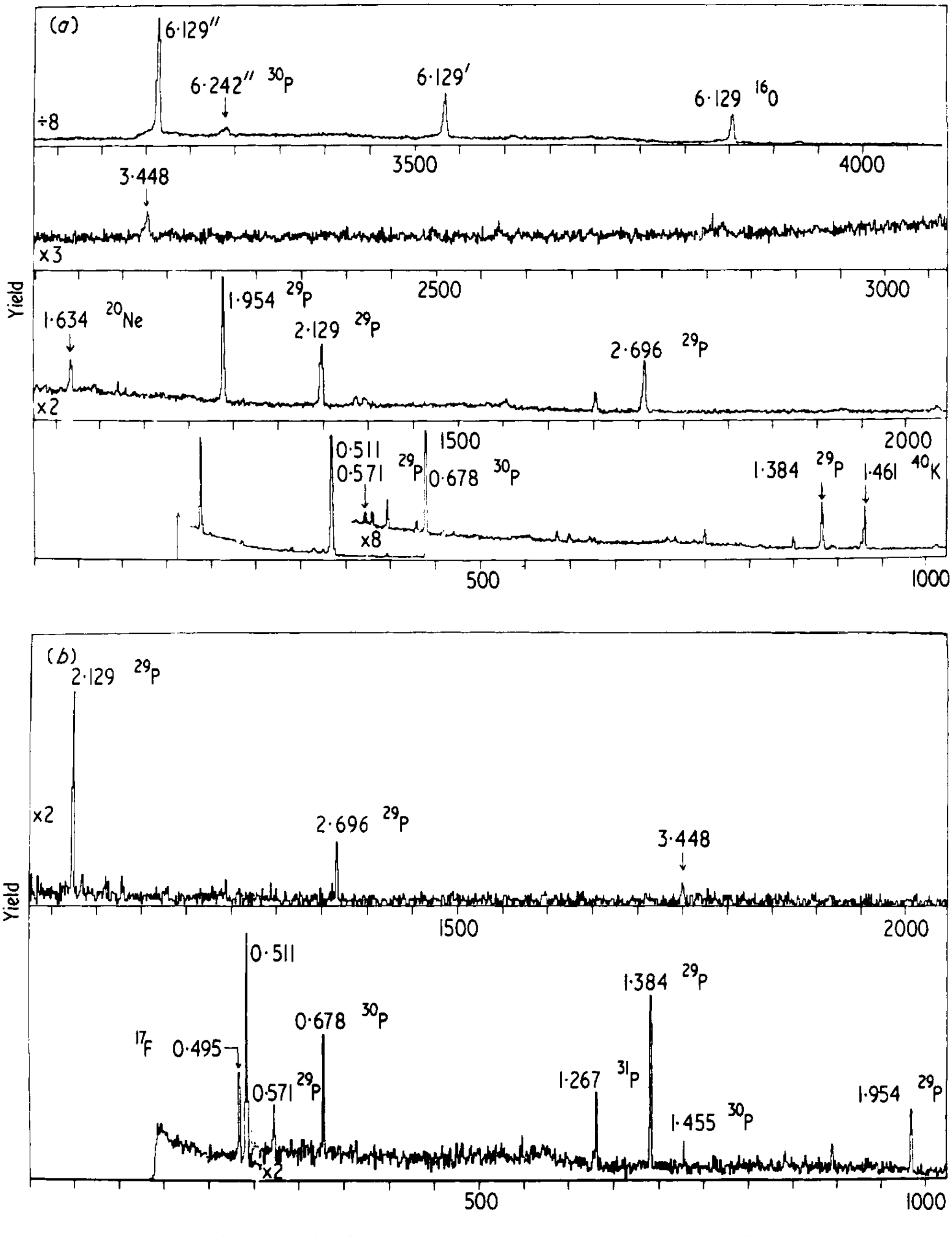

Figure 2. (*a*) Singles Ge(Li) γ ray spectrum taken from a natural silicon target at a proton energy of 1·381 MeV. (*b*) Ge(Li) spectrum of γ rays in coincidence with γ rays of 1·0 to 3·2 MeV recorded in a 7·5 × 7·5 cm NaI detector.

respectively. Monahan *et al* (1970) reported lines at 0·978 MeV and 1·723 MeV and assigned these to a decay from the resonance level to the fourth excited state and thence to the first excited state. These lines were not observed in either the singles spectra or in a spectrum taken in coincidence with a 1·0 to 3·2 MeV window in the NaI spectrum (figure 2(*b*)). If such a branch does exist then these data indicate that the branching ratio for the transition is less than 2 % of the total decay from the resonance level. This branch was not observed by Williams *et al* (1970) either.

A line at 3·448 MeV was present but the possibility of this being due to the decay of the fifth excited state, which has an excitation energy of 3·446 ± 0·016 MeV, can be discounted for the following reasons. There was no line equivalent to the necessary 0·633 MeV transition from the resonance level to the fifth excited state, the expected large branch from the 3·446 MeV level to the second excited state was not seen and the coincidence spectrum in figure 2(*b*) shows the line to be in coincidence with the γ rays of 1·0–3·2 MeV, when the largest possible coincidence γ ray could be 0·633 MeV.

A careful measurement (see § 3.5) of the energy of the 0·571 MeV γ ray shows that it fits the transition between the second and first excited states to within an experimental error of less than 0·4 keV. As this line is also enhanced in the coincidence spectrum we conclude that it is due to a branch in the decay of the second excited state. This transition was not reported by either Williams *et al* (1970) or Ejiri *et al* (1964a).

Branching ratios were calculated from the angular distribution normalization coefficients for the four strong lines and from the sum of singles spectra taken at a range of angles for the 0·571 MeV line. These are given in table 1.

The angular distribution of the 4·08 MeV to 1·38 MeV transition is shown in figure 3(*a*). Ejiri *et al* (1964a) limited the spin of the 4·08 MeV level to either $\frac{5}{2}$ or $\frac{7}{2}$ and our distribution confirms the $J = \frac{7}{2}$ assignment of Monahan *et al* (1970). The distribution of the 1·38 MeV γ rays is shown in figure 3(*b*) along with the χ^2 plots for $J(4{\cdot}08) = \frac{7}{2}$ and $\frac{5}{2}$. The full curve is the fit to the raw data and the broken curve the fit after allowing for the effect of the 8 % branch from the second excited state. The $\delta(1{\cdot}38)$ mixing ratios are in good agreement with the less accurate value obtained at the $E_p = 1{\cdot}65$ MeV resonance. The angular distribution of the 2·13 MeV γ rays confirms the assigned (Endt and Van der Leun 1967) value of $J = \frac{5}{2}$ for the second excited state. The angular distribution of the 1·95 MeV γ rays produced two possible values for the mixing ratio, the higher one of which is inconsistent with the lifetime (see § 3.6) of the state. The allowed mixing ratios are given in table 1.

3.4. The $E_p = 2080$ keV resonance

Ejiri *et al* (1964a) reported the direct ground-state transition as the only means of decay from the 4·761 MeV resonance level.

A singles Ge(Li) spectrum taken using a 9 keV thick natural silicon target is shown in figure 4(*a*). That the direct ground-state transition is not the only means of decay is indicated by a line at 1·384 MeV. The combinations of γ ray energies which sum to the resonance state energy are 1·384 MeV with 3·38 MeV, and 2·423 MeV with 2·34 MeV. Spectra taken on and below resonance show these lines to be resonant at this energy. As the 3·38 MeV and 2·34 MeV lines are approximately the same width as the primary ground-state transition (the 2·34 MeV full energy peak and the 3·38 MeV second-escape peak overlap each other if a target corresponding to the full width at half height of the resonance is used), these indicate transitions to the first excited state and to a level at 2·423 MeV.

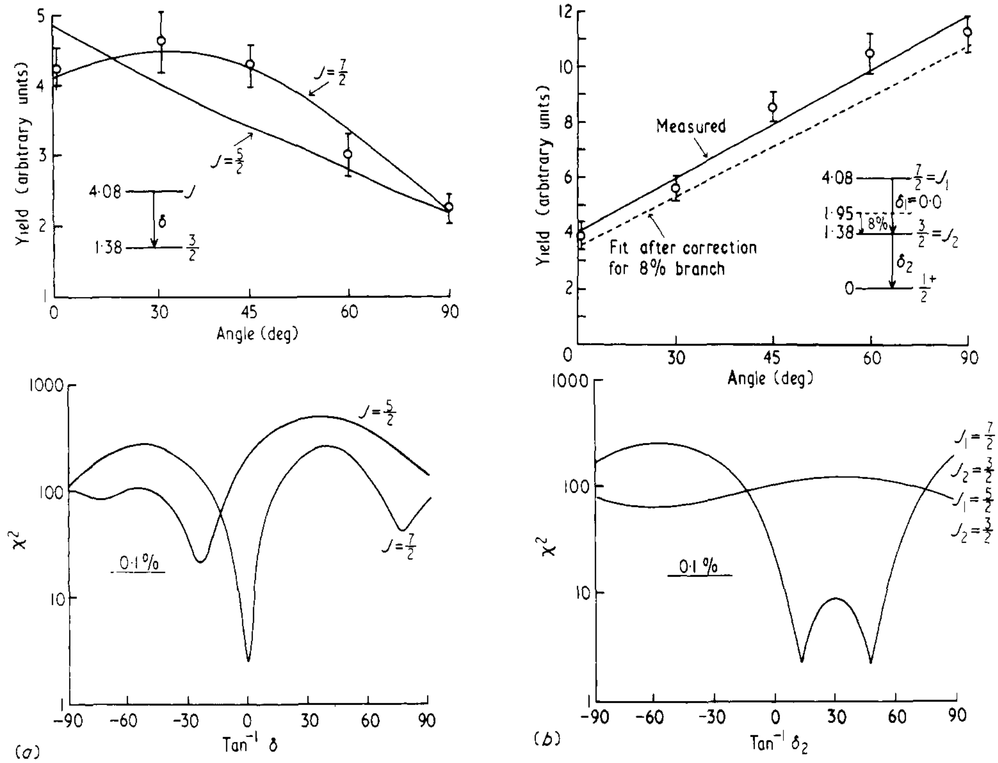

Figure 3. Angular distribution and χ^2 against δ plot of (*a*) 2·7 MeV γ rays from the transition from the sixth to the first excited states at the E_p = 1·381 MeV resonance and (*b*) 1·38 MeV γ rays depopulating the first excited state at the E_p = 1·381 MeV resonance.

A level at 2·424 ± 0·012 MeV has been investigated recently by Forster *et al* (1971) and Lamaze *et al* (1970) using the ^{32}S(p, α) ^{29}P reaction and also by Calvert and Joy (1970) using the ^{28}Si(d, n) ^{29}P reaction. Gamma decay involving this level has never been observed following the ^{28}Si(p, γ) ^{29}P reaction. There is a large discrepancy in the branching ratios for this state between the two (p, α) experiments and the (d, n) experiment. Despite the discrepancies, it would appear that if the 2·423 MeV γ ray is due to the decay of the third excited state of ^{29}P, then we should observe a branch of 1·039 MeV to the first excited state. This branch could not be observed in the singles spectra because of the high background due to the 1·274 MeV peak from the ^{29}Si(p, p′) ^{29}Si reaction.

A Ge(Li) spectrum taken in coincidence with a 1·39 MeV to 3·7 MeV NaI window (figure 4(*b*)) shows a line at 1·039 MeV as well as several lines from the ^{29}Si(p, γ) ^{30}P resonance at 2·070 MeV (Din and Davies 1971) excited owing to the use of a thicker target. We, therefore, conclude that there is a branch from the 4·76 MeV eighth excited state to the third excited state at 2·423 MeV, giving the decay scheme shown in figure 5. Branching ratios of the 2·423 MeV state were obtained from the coincidence spectrum in which the lower level of the NaI window was placed at 1·39 MeV to eliminate the second-escape peak of the 3·38 MeV line which is in coincidence with the 1·38 MeV line. The branching ratios are give in table 1.

Preliminary measurements of the angular distribution of the 4·76 MeV γ rays indicated that the distribution did not have the isotropy to be expected from the $J = \frac{1}{2}$

Table 1. Properties of the first, second, third, sixth, seventh and eighth excited states of ^{29}P

State	Energy (keV)	Spin and parity	Lifetimes (fs)	Transition	BR (%)	Mixing ratio	$\lvert M(\mathrm{M1})\rvert^2$ (Wu)	$\lvert M(\mathrm{E2})\rvert^2$ (Wu)
1st	1383·54±0·12	$\frac{3}{2}^{+}$	190±20	1·38 → 0	100	0·22±0·08	$(6·1\pm0·7)\times10^{-2}$	8^{+7}_{-3}
2nd	1953·96±0·20	$\frac{5}{2}^{+}$	340±90	1·95 → 0	92±4	E2		15±4
				1·95 → 1·38	8±4	0·14±0·09†	$(2·2\pm1·2)\times10^{-2}$	4^{+5}_{-2}
3rd	2423·5±0·9	$\frac{3}{2}^{+}$	70±35	2·42 → 0	80±10	−0·23±0·03	$(2·5\pm1·7)\times10^{-2}$‡	$1·1^{+1·1}_{-0·6}$‡
		(assumed)				or		
						3·3±0·3	$(2·3^{+2·1}_{-1·1})\times10^{-3}$	20^{+15}_{-10}‡
				2·42 → 1·38	20±10	0·15±0·14†	$(5\pm4)\times10^{-2}$‡	6^{+9}_{-5}‡
				2·42 → 1·95	<10		$\leqslant(1·4^{+1·5}_{-1·0}\times10^{-1})$‡	
6th	4080·6±0·3	$\frac{7}{2}^{+}$	13±5	4·08 → 1·38	44±3	E2		3·0±1·3
								or
								35±10
				4·08 → 1·95	56±3	0·11±0·02	$(8\pm4)\times10^{-3}$	$0·14^{+0·14}_{-0·05}$
							or	or
							0·13±0·05	$1·6^{+1·5}_{-0·5}$
7th	4343·7±2·1	$\frac{3}{2}^{+}$		4·34 → 0	92±2	E1		
				4·34 → 1·38	8±2	$\lvert\delta\rvert<0·04$		
8th	4760·7±2·0	$\frac{1}{2}^{+}$		4·76 → 0	90±3			
				4·76 → 1·38	7±3			
				4·76 → 2·42	3±2			

† Mixing ratios from Forster *et al* (1971).

‡ Using branching ratios from Forster *et al* (1971).

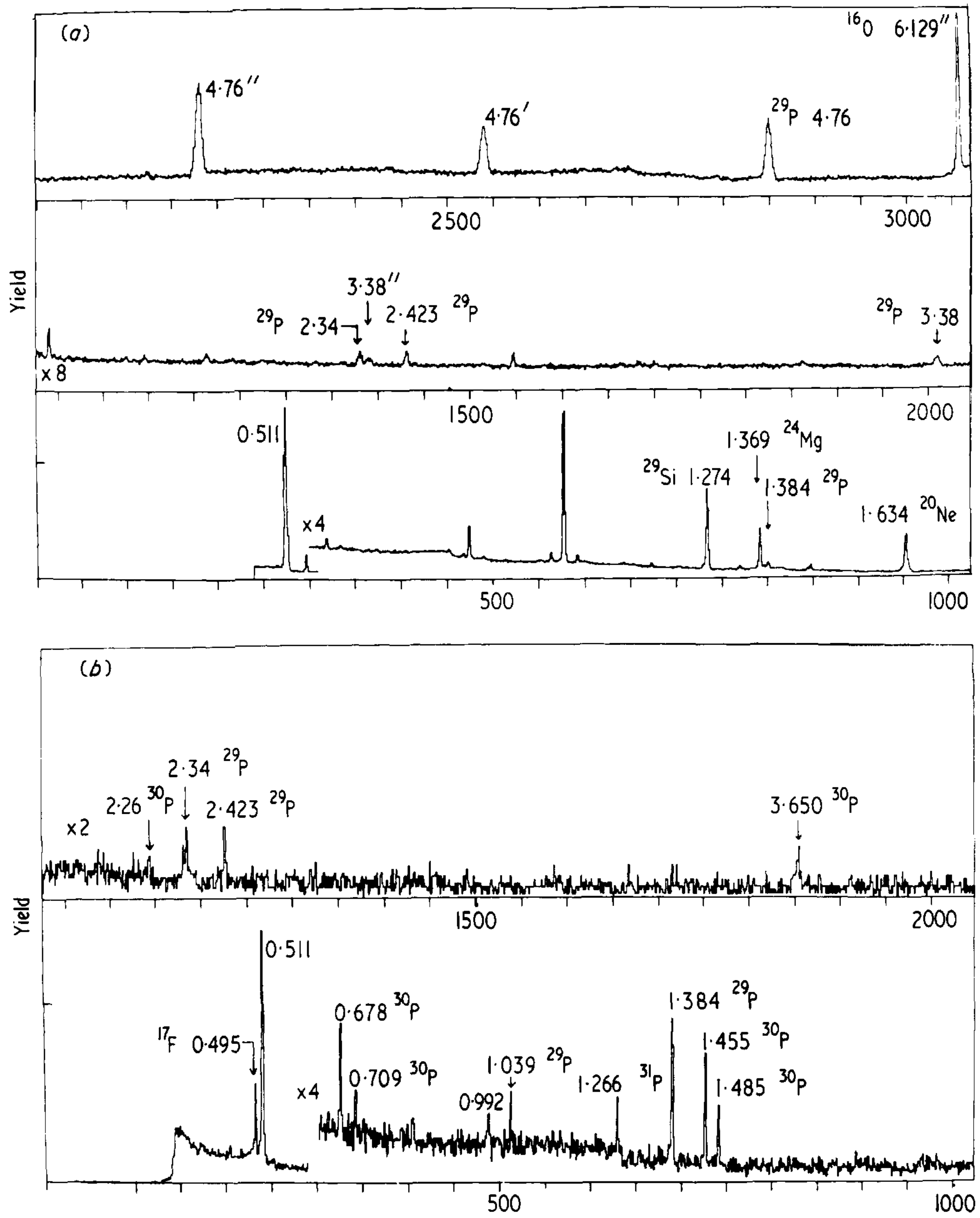

Figure 4. (*a*) Ge(Li) singles spectrum taken from a 9 keV thick natural silicon target at the E_p = 2·08 MeV resonance and (*b*) a Ge(Li) spectrum of γ rays in coincidence with γ rays of 1·39 to 3·7 MeV recorded in a 7·5 × 7·5 cm NaI detector.

assignment from elastic proton scattering by Vorona *et al* (1959). Our measurements, made between 0° and 90°, also showed that the distribution was represented better by a function including odd-order Legendre polynomials rather than one restricted to even orders. It was also noticed that the lineshapes appeared to have a low energy tail at all angles. This resonance was, therefore, investigated in more detail.

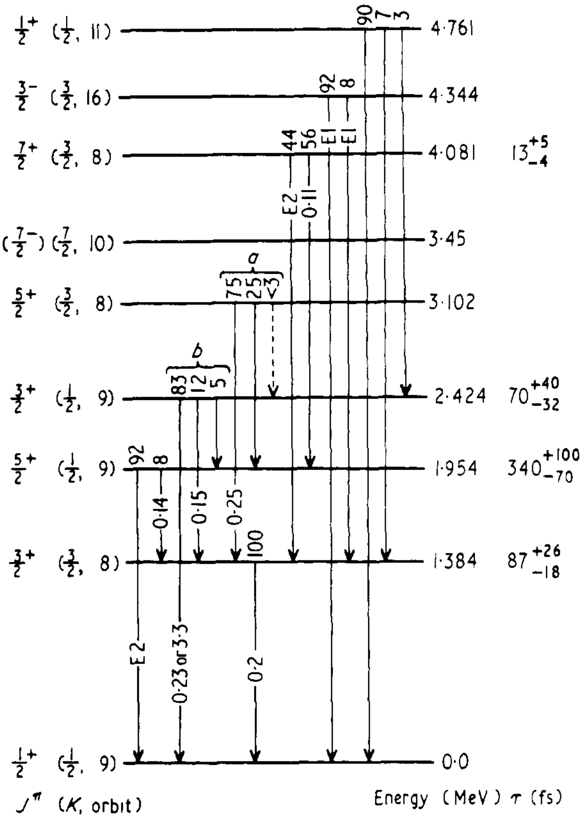

Figure 5. Summary of the properties of the first nine states of ^{29}P. *a* Endt and Van der Leun (1967); *b* Forster *et al* (1971).

A thin-target (2·5 keV) Ge(Li) detector yield curve was measured (figure 6). The proton energy scale was calibrated from the energy of the observed γ rays, using the Q value for the reaction determined to ±2 keV at the narrow $E_p = 1381$ keV resonance. The resonant energy, excitation energy and the total width at half height were found to be 2·086±0·002 MeV, 4·761±0·002 MeV and 16±3 keV, respectively. These values are in good agreement with those of Ejiri *et al* (1964a). The yield curve, however, shows a low energy tail similar to that observed in the 4·76 MeV γ ray lineshape from a thick target.

To investigate the possibility of this lineshape being the result of two overlapping resonances, spectra were accumulated at the three proton energies marked A, B and C in figure 6, to see if there was any change in the branching ratios across the yield curve. Owing to the smallness of the branches, which are only 7 % to the first excited state and 4·5 % to the third excited state, no significant change in these ratios was measured at the energies used.

In view of the preliminary indications of the presence of an odd Legendre polynomial coefficient, the angular distribution was remeasured over the range 0° to 145° using a 70 cm^3 detector. A 400 μg cm^{-2} (45 keV) target was used with a proton energy of 2·100 MeV. The angular distribution obtained (figure 7(*a*)) was fitted with a function of the form $W(\theta) = \Sigma_K A_K P_K(\cos\theta)$ with $K = 0, 1, 2, 3$. The distribution was well represented by

$$W(\theta) = 1 + (0{\cdot}334 \pm 0{\cdot}012)P_1(\cos\theta).$$

The inclusion of a P_2 term did not produce any significant improvement in the fit.

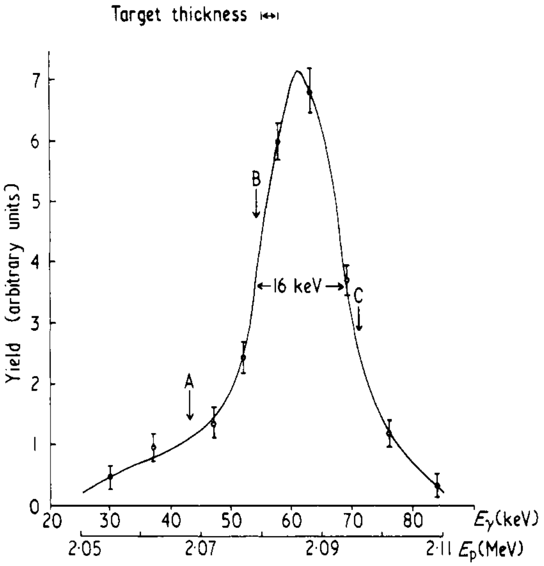

Figure 6. The yield of direct ground state transition γ rays in the region of the E_p = 2·08 MeV resonance.

Having established that the distribution was of the form $W(\theta) = 1 + A_1 \cos\theta$, the anisotropy between 0° and 140° was measured using a thin (2·5 keV) target at the peak of the resonance and at proton energies 8 keV above and below the peak. This gave A_1 coefficients of $0{\cdot}15 \pm 0{\cdot}05$, $0{\cdot}01 \pm 0{\cdot}05$ and $0{\cdot}38 \pm 0{\cdot}06$ respectively. The angular distribution of the 2·42 MeV and 1·38 MeV γ rays, which are fed from the resonance level, were measured using the 400 μg cm^{-2} target (figure 7(*b*)) and were found to be consistent with the isotropic distribution expected for the decay of a level populated via a state with spin $\frac{1}{2}$. The implications of these results are discussed in § 4.1.

3.5. *Measurements of excitation energies and Q value*

Spectra at the three resonances were accumulated at 90° to the beam using the full range of the 8192 channel ADC. Calibration peaks were accumulated simultaneously from sources of ^{60}Co, ^{22}Na, ^{137}Cs and Rd–Th. The spectra were analysed with the aid of a computer program which performed a background subtraction by fitting a quadratic function to the channels on each side of the peak and then calculated the centroid of the counts remaining after background subtraction. A calibration curve of quadratic form was fitted by the method of least squares to the centroid positions of the calibration peaks and the annihilation peak.

The weighted-mean values of the excitation energies of the levels observed, after correction for recoil, are given in table 1. Using the proton resonance energies of Endt and Van der Leun (1967) for the sixth, seventh and eighth excited states with our values of the excitation energies of these states, we obtain a weighted-mean Q value for the ^{28}Si(p, γ) ^{29}P reaction of $2747{\cdot}1 \pm 1{\cdot}7$ keV, in good agreement with the value $2745{\cdot}0 \pm 5{\cdot}0$ keV of Monahan *et al* (1970).

3.6. *Lifetime measurements*

Lifetimes of the 1·384, 1·954, 2·423 and 4·081 MeV states were measured with the Doppler

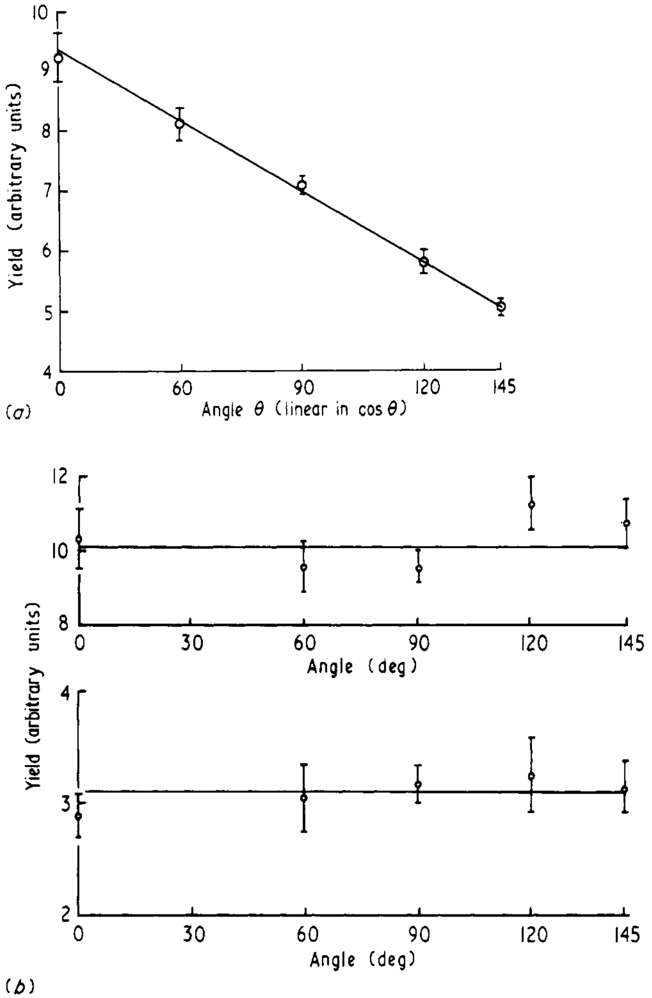

Figure 7. (*a*) Angular distribution of 4·76 MeV γ rays depopulating the eighth excited state at the E_p = 2·08 MeV resonance. The fit is $W(\theta) = 7{\cdot}023\{1+(0{\cdot}334 \pm 0{\cdot}012)P_1(\cos\theta)\}$. (*b*) Angular distributions of γ rays depopulating the first and third excited states at the E_p = 2·08 MeV resonance.

shift attenuation method (DSAM). The theoretical values of the attenuation coefficient $F(\tau)$ were calculated using the method of Blaugrund (1966) with the electronic stopping powers of Lindhard *et al* (1963) and the expression by Engelbertink *et al* (1968) to approximate the nuclear stopping power. Lifetimes were then determined by comparison of the experimentally determined values of F with the theoretically calculated $F(\tau)$ curves. The energies of the centroids of the γ ray lines at different angles were measured as described in § 3.5. The 1·384 MeV state lifetime was measured at the E_p = 1381 keV resonances, the 1·954 MeV and 4·081 MeV states at the E_p = 1381 keV resonance, and the 2·423 MeV state at the E_p = 2080 keV resonance. The targets consisted of a range of thicknesses of natural silicon evaporated onto gold backing as indicated below.

Because the E_p = 1650 keV resonance has a width of 56 keV (Endt and Van der Leun 1967) a proton energy corresponding to 15 keV below the peak of the resonance was used, so that only a small proportion of the interaction took place within the recoiling distance ($\sim 25\,\mu g\,cm^{-2}$) from the gold backing. As a check on this, attenuation factors were measured using $220\,\mu g\,cm^{-2}$ and $350\,\mu g\,cm^{-2}$ thick targets and as there was no significant difference between these results we conclude that only an insignificant number of ^{29}P recoils reached the gold backing.

The E_p = 1381 keV resonance has a width of only 3 keV, thus there was no problem about preventing the recoiling ^{29}P ions from reaching the gold backing. The resonance was first located using a $20\,\mu g\,cm^{-2}$ target and then a $220\,\mu g\,cm^{-2}$ target was substituted for the DSAM measurements. As the resonance level has a finite lifetime this must be taken into account when calculating the theoretical $F(\tau)$ value for the 1·384 and 1·954 MeV γ ray transitions. This was done with the aid of the formulae in Piluso *et al* (1969). The analysis for the 1·384 MeV γ ray was complicated by the decay from the direct capture ^{16}O(p, γ) ^{17}F reaction, as discussed by Joy and Barnes (1971). This tended to make the DSAM measurement less accurate at this resonance energy than at the E_p = 1650 keV resonance, although good agreement was found between the two results.

The E_p = 2080 keV resonance, which has a width of 16 keV, was first located using the $20\,\mu g\,cm^{-2}$ target and then the $350\,\mu g\,cm^{-2}$ target was substituted for the DSAM measurements. The Doppler shift of the direct ground-state transition from the resonance level was measured and found to be consistent with a full shift.

The experimental shifts and $F(\tau)$ values for these resonances are listed in table 2, and the resulting weighted mean lifetime values for the states are given in table 1.

Table 2. Experimentally determined Doppler shift attenuation factors

E_p (MeV)	E_γ (MeV)	$F(\%)$
1·65	1·38	37 ± 3
1·38	2·70	91 ± 7
	2·13	96 ± 9
	1·95	19 ± 4
	1·38	33 ± 6
2·08	2·42	69 ± 13

4. Discussion and results

4.1. The angular distribution of the 4·761 MeV γ rays

The presence of an odd Legendre polynomial coefficient could be the result of either the $\frac{1}{2}^+$ resonance state having a significant amplitude of the 'wrong' parity wavefunction or there being a mixture of spin values. The first of these possibilities can be eliminated as it would require an E1/M1 mixing ratio of δ = 5·9 or 0·17 to produce the observed A_1 coefficient and it is difficult to see how such a large amplitude of the 'wrong' parity wavefunction could exist. The ratio of the amplitude of the irregular admixture due to the weak interaction to the regular amplitude in the nuclear wavefunction is estimated to be about 10^{-7} (Blin-Stoyle 1960). Thus, even allowing for extreme cases of M1 retardation and E1 enhancement and for the fact that E1 transitions are naturally faster than M1 transitions, there is still a factor of the order of 10^4 in relative amplitude which cannot be accounted for in this way. We are thus left with the possibility of a mixture of spin values.

If the observed distribution is the result of interference between the resonance state and the tail of some other resonance, there is only one known $\frac{1}{2}^-$ state in ^{29}P within the range of interest. A broad strong resonance in the ^{28}Si(p, p) ^{28}Si reaction was reported by Vorona *et al* (1959) at a proton energy of 2·90 ± 0·06 MeV. The elastic scattering data showed that the resonance was due to $l = 1$ formation of a $\frac{1}{2}^-$ state in ^{29}P with an estimated width of $\Gamma_l = 450 \pm 150$ keV. The relevant parameters of this resonance are shown in row B of table 3.

Table 3. The excitation energies E_J, proton widths Γ_l and γ widths Γ_L, of the levels used to calculate the angular distribution of the 4·76 MeV γ rays

	Level J^π	E_J (MeV)	Γ_l (keV)	Γ_L (eV)
A	$\frac{1}{2}^+$	4·761†	16†	0·33‡
B	$\frac{1}{2}^-$	5·53§	425§	<2·0‡
C	$\frac{1}{2}^-$	4·735	15	0·03

† This work. ‡ Wozniak *et al* (1969).
§ Spear *et al* (1971).

Alternatively, the experimental yield curve with its low energy tail suggests the presence of two unresolved resonances. In this case, it is possible to reproduce the observed yield curve by assuming that there is a small $\frac{1}{2}^-$ resonance centred at a proton energy of 4·735 MeV and having the parameters listed in row C of table 3.

The experimental A_1 coefficients determined at three energies using a thin target are compared in figure 8 with the theoretical values calculated under the two alternative assumptions given above. As the gamma width of the 5·53 MeV state is not known, these calculated A_1 coefficients are an upper estimate produced by using the upper limit for Γ_L given by Ejiri *et al* (1964a).

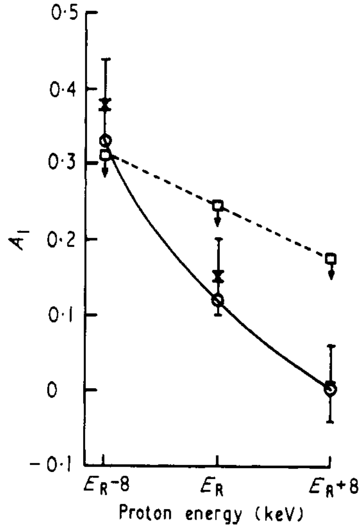

Figure 8. Theoretical and experimental values of the A_1 angular distribution coefficient of 4·76 MeV γ rays as a function of proton energy near the E_p = 2·08 MeV resonance. ○, calculated assuming a doublet; □, calculated using 5·53 MeV $\frac{1}{2}^-$ state; ×, experiment.

As can be seen from figure 8, neither of the two possibilities is completely inconsistent with the observed anisotropy. However, it does seem that the hypothesis of an unresolved doublet produces a better fit both in its absolute magnitude of the A_1 coefficients and its rate of change with energy across the 4·761 MeV resonance. On the other hand, the existence of two $\frac{1}{2}^-$ states in ^{29}P below an excitation energy of 6 MeV is not supported by comparison with neighbouring nuclei, nor by theoretical predictions.

We cannot, therefore, come to any firm conclusion about the origin of the observed A_1 coefficient in the angular distribution of the 4·67 MeV γ rays, except to say that there seems no reason to doubt the assigned J^π value of $\frac{1}{2}^+$ for the eighth (ninth?) excited state.

4.2. *Spins, parities, mixing ratios and transition rates*

4.2.1. The first excited state (1385 keV). The value of 187^{+26}_{-18} fs for the lifetime of the state, which is equivalent to a γ width of $\Gamma_\gamma = 3{\cdot}5 \pm 0{\cdot}4$ meV is in good agreement with the values of 190 ± 60 fs (Monahan *et al* 1970), 210 ± 60 fs (Cummings and Donahue 1970) and 180 ± 30 fs (Bizetti *et al* 1969). The angular distribution measurements give $J = \frac{3}{2}$ with $\delta = 0{\cdot}222$ or $1{\cdot}09$. Negative parity for the state can be excluded as this would entail M2 enhancements of greater than 10 Wu and the $\delta = 1{\cdot}09$, positive parity combination can also be eliminated as this would require an E2 enhancement of 91 Wu. These results are summarized in table 1 and are in substantial agreement with those of Monahan *et al* (1970) and Williams *et al* (1970).

4.2.2. The second excited state (1954 keV). The mean life of 340^{+100}_{-70} fs is consistent with the values of 390^{+190}_{-100} fs of Williams *et al* (1970) and 370 ± 80 fs of Monahan *et al* (1970) and the angular distribution measurements confirm the assigned $J = \frac{5}{2}$ spin value. The possibility of negative parity can be eliminated as this would require M2 enhancements of greater than 40 Wu for the ground state transition and thus also the $\delta = 3{\cdot}73$ possibility can be excluded leaving this as a pure E2 transition to the ground state.

The 0·571 MeV branch to the first excited state was too weak to allow an angular distribution measurement. Our value of $(8 \pm 4)\%$ for this branch intensity is in good agreement with the value of $(7 \pm 3)\%$ (Lamaze *et al* 1970) and $(9 \pm 2)\%$ Forster *et al* (1971) and in disagreement with the value of $(38 \pm 5)\%$ (Calvert and Joy 1970). Forster *et al* find a mixing ratio of $\delta(\mathrm{E2/M1}) = -0{\cdot}14 \pm 0{\cdot}09$ for the transition and this was used to calculate the M1 and E2 strengths given in table 1.

4.2.3. The third excited state (2423 keV). Our value of 70^{+40}_{-32} fs is the first reported value for the lifetime of this state. The branching ratios to the ground, first and second excited states were found to be $(80 \pm 10)\%$, $(20 \pm 10)\%$ and less than 10% respectively, which are in substantial agreement with the more accurate values of Forster *et al* (1971) and Lamaze *et al* (1970). The averages of their values obtained via the ^{32}S(p, α) ^{29}P reaction are $(85 \pm 3)\%$, $(12 \pm 2)\%$ and $(3 \pm 2)\%$ and these values were used to calculate the transition strengths given in table 1.

This level is fed by γ decay only from the spin $\frac{1}{2}$, 4·761 MeV state, hence it was not possible to extract any mixing ratio information from the angular distribution measurements and so the values given by Forster *et al* were used to calculate the multipole enhancements given in table 1.

4.2.4. The fifth excited state. The values of $\Gamma_\gamma < 60$ µeV for the $E_\mathrm{p} = 720$ keV resonance improves upon the upper limit values of 0·5 meV of van Oostrum *et al* (1961), 6×10^{-4} eV of Okano *et al* (1960) and 9×10^{-5} eV of Zuk *et al* (1971). The lifetime of the fifth excited state of ^{29}Si has been measured by Wozniak *et al* (1969) as $4{\cdot}8^{+3{\cdot}0}_{-1{\cdot}5}$ ps, which implies an E1 retardation of $5{\cdot}5 \times 10^{-5}$ Wu for the transition to the second excited state. As our value of $\Gamma_\gamma \leqslant 6 \times 10^{-5}$ eV is equivalent to an E1 retardation for the same transition of 3×10^{-5} Wu it would seem that a further refinement of the experimental method might succeed in detecting this resonance. An experiment along these lines is in progress at the present time.

4.2.5. The sixth excited state (4081 keV). The $E_p = 1381$ keV resonance was first investigated by Ejiri *et al* (1964a) using NaI detectors and has been re-examined recently with Ge(Li) detectors by Monahan *et al* (1970), Williams *et al* (1970) and others. The strength $(2J+1)\Gamma_p\Gamma_\gamma/\Gamma$ of the resonance has been measured as (16 ± 8) meV (Ejiri *et al* 1964a) and (28 ± 6) meV (Zuk *et al* 1971), giving a weighted mean value of (24 ± 5) meV, Our value of 13^{+5}_{-4} fs for the lifetime of the resonance state is equivalent to a width of 51^{+20}_{-14} meV. Assuming $\Gamma = \Gamma_p + \Gamma_\gamma$ gives the two roots of the quadratic equation for Γ_γ of 47 ± 13 meV or 3 ± 1 meV. As we expect $\Gamma_p \gg \Gamma_\gamma$, the smaller of these two results is the more probable but, as there is no other reason for rejecting the larger value, both possibilities are included in the calculated transition strengths given in table 1. The fact that the mirror level in ^{29}Si is reported (Wozniak *et al* 1969) to have a width $\Gamma_\gamma = 3 \pm 1$ meV lends support to the lower value of $\Gamma_\gamma = 3 \pm 1$ meV for the $E_p = 1{\cdot}381$ meV resonance in ^{29}P.

4.2.6. The seventh and eighth excited states (4344 and 4761 keV). The properties of the seventh excited state were found to be in substantial agreement with those reported by Ejiri *et al* (1964a). The distribution of the ground state transition γ rays from the 4·761 MeV state has already been discussed in § 4.1. Branching ratios for these states are given in table 1.

4.3. *Comparison with model predictions*

Both the weak coupling approach (Ejiri 1964, Bailey and Choudhury 1970) and the strong coupling Nilsson model approach (Bromley *et al* 1957, Ejiri 1964) have been employed to try to calculate the properties of ^{29}P and ^{29}Si. Bromley *et al* found that the Nilsson model could explain many of the then-known properties of ^{29}Si if an oblate deformation of $\delta = -0{\cdot}15$ was assumed. The need for a negative deformation is most strikingly demonstrated by the appearance at low energy (3·62 MeV) of a $\frac{7}{2}$ level which is explained in terms of the odd particle being in Nilsson orbit 10. This view is supported by the recent work of Spear *et al* (1971) and Bardin *et al* (1970) who discuss the possibility of a $\frac{9}{2}$ level at 5·256 MeV in ^{29}Si being the second member of the $K = \frac{7}{2}$ band based on the 3·62 MeV level.

A negative deformation of $\eta = -3$, $\delta = -0{\cdot}15$, was adopted by Ejiri (1964) as the basis for his strong coupling calculation for ^{29}P. Using rotation–vibration and rotation–particle coupling of bands based on Nilsson orbits 8, 9 and 11, Ejiri reproduced the positive-parity experimental energy-level spectrum up to an energy of approximately 5 MeV. The negative-parity states at 3·45 MeV (assumed $\frac{7}{2}$) and 4·34($\frac{3}{2}$) were assumed to be the intrinsic states of Nilsson orbits 10 and 16, respectively. The latter assignment is supported by the large proton reduced width reported by Calvert and Joy (1970) suggesting that this level corresponds to a $2p_{3/2}$ configuration. The lower energy levels of ^{29}P are shown in terms of Ejiri's K band assignments in figure 5. Similarly the band-mixing calculations of Hirko (1970) (see Bardin *et al* 1970) account very well for properties of positive-parity states with $E_x \leqslant 4·07$ MeV in ^{29}Si, again with oblate deformation. An oblate structure for the potential in which the non-core particles move is also suggested by the evidence for the presence of a ground state rotational band of ^{28}Si (Gibson *et al* 1968) and with a negative deformation parameter as determined by the reorientation effect in Coulomb excitation.

The weak-coupling calculations of Bailey and Choudhury (1970) and Ejiri (1964) both took into their account single-particle $1d_{3/2}$ and $2s_{1/2}$ states coupled to up to three quadrupole phonons. Within this framework they were only able to reproduce the

Table 4. A comparison of experimental data from the present work with theoretical values obtained by Bailey and Choudhury (1970) using the vibrational model and Ejiri *et al* (1964b) using both the vibrational model and the rotational model. 4(*a*) compares branching ratios, 4(*b*) compares lifetimes, 4(*c*) compares gamma ray widths

(*a*) Branching ratios (%)

Transition	Bailey and Choudhury (vib)	Ejiri *et al* (rot)	Ejiri *et al* (vib)	Experiment
1·95 → 0	74	43	63	92 ± 4
1·95 → 1·38	26	57	37	8 ± 4
2·42 → 0	23			85 ± 3
2·42 → 1·95	21			3 ± 2
2·42 → 1·38	56			12 ± 2
4·08 → 1·38		92	99	44 ± 3
4·08 → 1·95		8	1	56 ± 3
4·34 → 0		96	98	92 ± 2
4·34 → 1·38		4	2	8 ± 2

(*b*) Lifetimes (fs)

Level (MeV)	Bailey and Choudhury (vib)	Ejiri *et al* (vib)	Ejiri *et al* (rot)	Experiment
1·38	2790			187 ± 20
1·95	410	175	678	340 ± 90
2·42	550			70 ± 35
4·08		41	270	13 ± 5

(*c*) Gamma ray widths (meV)

Transition (MeV)	Bailey and Choudhury		Ejiri *et al* (vib)		Ejiri *et al* (rot)		Experiment	
	E2	M1	E2	M1	E2	M1	E2	M1
1·38 → 0·0	$2·35 \times 10^{-1}$	$1·57 \times 10^{-4}$					$1·6 \times 10^{-1}$	3·36
1·95 → 0·0	1·20		2·4		0·42		1·78	
1·95 → 1·38	$6·71 \times 10^{-5}$	$4·29 \times 10^{-1}$		1·6		0·55	3×10^{-3}	$1·6 \times 10^{-1}$
2·42 → 0·0	$2·74 \times 10^{-1}$	$6·38 \times 10^{-4}$					4×10^{-1}	7·6
2·42 → 1·38	$7·50 \times 10^{-3}$	$6·65 \times 10^{-1}$					2×10^{-2}	1·1
2·42 → 1·95	$1·77 \times 10^{-6}$	$2·42 \times 10^{-1}$					$\leqslant 2·7 \times 10^{-1}$	$\leqslant 2·7 \times 10^{-1}$
4·08 → 1·38			16		2·2		0·9 or 22	
4·08 → 1·95				0·026		0·19	0·02 or 0·3	1·1 or 27

correct positive-parity spin sequence up to the third excited state. Above this they both predicted a $\frac{7}{2}$ state to be lower than the next $\frac{5}{2}$ state.

The absolute and relative transition strengths predicted by both models are in poor agreement with the experimental values, mainly as a result of their inability to calculate the M1 transition strengths, which are often predicted to be as much as 10^4 times too small by the weak coupling model. Ejiri only published predictions for the few transition rates which were known at that time so a complete comparison with the transition strengths predicted by the strong coupling model is not available. Experimental and theoretical branching ratios are compared in table 4(*a*), lifetimes in table 4(*b*) and absolute γ ray widths in table 4(*c*). From these it can be seen that although the M1 strengths are generally poorly predicted, the E2 strengths are generally correct to within an order of magnitude for both models.

5. Conclusion

It would seem that neither the rotational model nor the vibrational model, as they have been applied to this case, is sufficiently intricate to account for the γ decay rates of ^{29}P which are now known. The Nilsson model produces a more accurate energy-level spectrum than either of the vibrational model calculations, the spins and parities of the first ten states being accounted for and all excitation energies fitted to within 0·5 MeV up to 5 MeV. The vibrational model only satisfactorily reproduces the first three excited states before large discrepancies occur. Castel *et al* (1970) reproduce the first six energy levels of ^{29}Si with reasonable values for the branching ratios of the first four states using a weak coupling model calculation which includes $d_{5/2}$ single-quasiparticle states and anharmonicity of the two phonon states. Castel and Johnstone (1971) also report greatly improved M1 strength calculations for ^{29}Si by making an allowance for magnetic dipole excitations as well as quadrupole excitations. It would obviously be of great interest to have these calculations repeated for ^{29}P. It is also important to have a much more comprehensive rotational model calculation including such features as hole bands for Nilsson orbits 5, 6 and 7, as this model appears to have fewer major discrepancies between theoretical predictions and experimental data and is thus probably a more promising basis for future theoretical calculations.

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